IN SEARCH OF THE SPACETIME TORSION

J. G. PEREIRA
Instituto de Física Teórica, Universidade Estadual Paulista
Rua Pamplona 145, 01405-900 São Paulo, Brazil

Whether torsion plays or not a role in the description of the gravitational interaction is a problem that can only be solved by experiment. This is, however, a difficult task: since there are different possible interpretations for torsion, there is no a model–independent way to look for it. In these notes, two different possibilities will be reviewed, their consistency analyzed, and the corresponding experimental outputs briefly discussed.

1 Gravitation and Universality

Gravitation has a quite peculiar property: particles with different masses and different compositions feel it in such a way that all of them acquire the same acceleration and, given the same initial conditions, follow the same path. Such universality of response — usually referred to as *universality of free fall* — is the most fundamental characteristic of the gravitational interaction.[1] It is unique, peculiar to gravitation: no other basic interaction of nature has it. Universality of free fall is usually identified with the weak equivalence principle, which establishes the equality between inertial and gravitational masses. In fact, in order to move along the same trajectory, the motion of different particles must be independent of their masses, which have then to be canceled out from the equations of motion. Since this cancellation can only be made when the inertial and gravitational masses coincide, this coincidence naturally implies universality.

Einstein's general relativity is a theory fundamentally based on the weak equivalence principle. In fact, to comply with universality, the presence of a gravitational field is supposed to produce *curvature* in spacetime, the gravitational interaction being achieved by letting (spinless) particles to follow the *geodesics* of the curved spacetime. In general relativity, therefore, geometry replaces the concept of gravitational force, and the trajectories are determined, not by force equations, but by geodesics. It is important to emphasize that only a universal interaction can be described by such a geometrization, in which the responsibility for describing the interaction is transferred to spacetime. It is also important to remark that, in the eventual lack of universality, the geometrical description of general relativity would break down.[2]

The fundamental connection of general relativity is the Christoffel connection, a Lorentz-valued connection written in a coordinate basis.[3] In terms of the spacetime[a] metric $g_{\mu\nu}$, it is written as

$$\overset{\circ}{\Gamma}{}^{\rho}{}_{\mu\nu} = \tfrac{1}{2}\, g^{\rho\lambda}(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\lambda\mu} - \partial_\lambda g_{\mu\nu}). \tag{1}$$

It is a connection with vanishing torsion, $\overset{\circ}{T}{}^{\rho}{}_{\nu\mu} = 0$, but non- vanishing curvature, $\overset{\circ}{R}{}^{\rho}{}_{\lambda\nu\mu} \neq 0$. In terms of this connection, the equation of motion of a test particle is given by the geodesic

[a] We use the Greek alphabet $\mu, \nu, \rho, \ldots = 0, 1, 2, 3$ to denote spacetime indices.

equation

$$\frac{du_\mu}{ds} - \overset{\circ}{\Gamma}{}^{\theta}{}_{\mu\nu}\, u_\theta\, u^\nu = 0, \tag{2}$$

which says that the particle four-acceleration vanishes identically. This property reveals the absence of the concept of gravitational force, a basic characteristic of the geometric description.

2 What About Torsion?

A general Lorentz connection has two fundamental properties: curvature and torsion.[4] Why should then matter produce *only curvature*? Was Einstein wrong when he made this assumption by choosing the Christoffel connection? Does torsion play any role in gravitation? Cartan was the first to ask these questions, soon after the advent of general relativity. As a possible answer, a new theory was formulated, called Einstein–Cartan theory,[5] in which the Christoffel connection was replaced by a general connection presenting both curvature and torsion.

The main idea behind the Einstein–Cartan construction is the fact that, at the microscopic level, matter is represented by elementary particles, which in turn are characterized by mass (that is, energy and momentum) and spin. If one adopts the same *geometrical spirit of general relativity*, not only mass but also spin should be source of gravitation at this level. Like in general relativity, energy and momentum should appear as source of curvature, whereas spin should appear as source of torsion. This means essentially that, in this theory, curvature and torsion represent independent degrees of freedom of gravity. As a consequence, there might exist new physics associated to torsion. Of course, at the macroscopic level, where spins vanish, the Einstein–Cartan theory coincides with general relativity. At the microscopic level, however, where spins are relevant, it shows different results from general relativity. If this is interpretation is correct, Einstein made a mistake when he did not include torsion in general relativity.

Because of the Einstein–Cartan theory, there is a widespread belief that torsion is intimately related to spin, and consequently important only at the microscopic level. This belief, however, is not fully justified: in addition to lack experimental support, it is based on a very particular model for gravitation, which is well known to present several consistency problems. For example, it is not consistent with the strong equivalence principle.[6] Another relevant problem is that, when used to describe the interaction of the electromagnetic field with gravitation, the Einstein–Cartan coupling prescription violates the U(1) gauge invariance of Maxwell's theory.[7] This last problem is usually circumvented by *postulating* that the electromagnetic field does not couple to torsion.[8] This "solution", however, is quite unreasonable.[9] The purpose of these notes is to call the attention to another possible interpretation for torsion,[6] which is consistent and does not present the above mentioned problems. This solution is based on a different model for gravitation, known as the *teleparallel gravity equivalent of general relativity*,[10] or simply *teleparallel gravity*.

3 A Glimpse on Teleparallel Gravity

Teleparallel gravity corresponds to a gauge theory for the translation group, with the field strength given by the torsion tensor. Its main difference in relation to general relativity is the connection field: instead of Christoffel, the fundamental connection of teleparallel gravity is the so called Weitzenböck connection. In terms of the tetrad[b] $h^a{}_\mu$, it is written as

$$\overset{\bullet}{\Gamma}{}^{\rho}{}_{\mu\nu} = h_a{}^\rho\, \partial_\nu h^a{}_\mu. \tag{3}$$

[b] We use the Latin alphabet $a, b, c, \ldots = 0, 1, 2, 3$ to denote algebraic indices related to the tangent Minkowski spaces. These indices are raised and lowered with the Minkowski metric η_{ab}, whereas the spacetime indices are raised and lowered with the metric $g_{\mu\nu} = \eta_{ab}\, h^a{}_\mu\, h^b{}_\nu$.

In contrast to Christoffel, it is a connection with non-vanishing torsion, $\overset{\bullet}{T}{}^{\rho}{}_{\nu\mu} \neq 0$, but vanishing curvature, $\overset{\bullet}{R}{}^{\lambda\rho}{}_{\nu\mu} = 0$. The Weitzenböck and the Christoffel connections are related by

$$\overset{\bullet}{\Gamma}{}^{\rho}{}_{\mu\nu} = \overset{\circ}{\Gamma}{}^{\rho}{}_{\mu\nu} + \overset{\bullet}{K}{}^{\rho}{}_{\mu\nu}, \tag{4}$$

where

$$\overset{\bullet}{K}{}^{\rho}{}_{\mu\nu} = \tfrac{1}{2}\left(\overset{\bullet}{T}{}_{\mu}{}^{\rho}{}_{\nu} + \overset{\bullet}{T}{}_{\nu}{}^{\rho}{}_{\mu} - \overset{\bullet}{T}{}^{\rho}{}_{\mu\nu}\right) \tag{5}$$

is the contortion tensor.

In teleparallel gravity, the equation of motion of a test particle is given by the *force equation*[11]

$$\frac{du_\mu}{ds} - \overset{\bullet}{\Gamma}{}^{\theta}{}_{\mu\nu}\, u_\theta\, u^\nu = \overset{\bullet}{T}{}^{\theta}{}_{\mu\nu}\, u_\theta\, u^\nu, \tag{6}$$

with torsion playing the role of *gravitational force*. It is similar to the Lorentz force equation of electrodynamics, a property related to the fact that, like Maxwell's theory, teleparallel gravity is also a gauge theory. Using expression (4), the force equation (6) can be rewritten in terms of the Christoffel connection, in which case it reduces to the geodesic equation of general relativity:

$$\frac{du_\mu}{ds} - \overset{\circ}{\Gamma}{}^{\theta}{}_{\mu\nu}\, u_\theta\, u^\nu = 0. \tag{7}$$

The force equation (6) of teleparallel gravity and the geodesic equation (7) of general relativity, therefore, describe the same physical trajectory. This means that the gravitational interaction has two *equivalent* descriptions: one in terms of curvature, and another in terms of torsion.[12] Although equivalent, however, there are conceptual differences between these two descriptions. In general relativity, curvature is used to *geometrize* the gravitational interaction. In teleparallel gravity, on the other hand, torsion accounts for gravitation, not by geometrizing the interaction, but by acting as a *force*. As a consequence, there are no geodesics in teleparallel gravity, but only force equations, quite analogous to the Lorentz force equation of electrodynamics.

One may wonder why gravitation has two equivalent descriptions. This duplicity is related precisely to that peculiarity: universality. Like the other fundamental interactions of nature, gravitation can be described in terms of a gauge theory – - just teleparallel gravity. Universality of free fall, on the other hand, makes it possible a second, geometrized description, based on the weak equivalence principle — just general relativity. As the sole universal interaction, it is the only one to allow also a geometrical interpretation, and hence two alternative descriptions. From this point of view, curvature and torsion are simply alternative ways of describing the gravitational field,[13] and consequently related to the same degrees of freedom of gravity. If this interpretation is correct, Einstein was right when he did not include torsion in general relativity.

4 Conclusions

According to the Einstein–Cartan theory, as well as to other generalizations of general relativity, torsion represents additional degrees of freedom of gravity. As a consequence, new physical phenomena associated to its presence are predicted to exist. With this point of view in mind, there has been recently a proposal to look for these new phenomena using the Gravity Probe B data.[14] The idea is that, if torsion is able to couple to spin, consistency arguments require that it might also be able to couple to rotation — that is, to orbital angular momentum. The data of Gravity Probe B could then be used to look for signs of this coupling. On the other hand, from the point of view of teleparallel gravity, torsion does not represent additional degrees of freedom, but simply an alternative to curvature in the description of gravitation. In this case, there are no new physical effects associated with torsion. According to teleparallel gravity, therefore, torsion has already been detected: it is the responsible for all known gravitational effects, including the

physics of the solar system, which can be reinterpreted in terms of a force equation, with torsion playing the role of gravitational force.

Which of these interpretations is the correct one? From the theoretical point of view, we can say that the teleparallel interpretation presents several conceptual advantages in relation to the Einstein–Cartan theory: it is consistent with the strong equivalence principle,[6] and when applied to describe the interaction of the electromagnetic field with gravitation, it does not violate the U(1) gauge invariance of Maxwell's theory.[9] From the experimental point of view, on the other hand, at least up to now, there are no evidences for new physics associated with torsion. We could then say that the existing experimental data favor the teleparallel point of view, and consequently general relativity. However, due to the weakness of the gravitational interaction, no experimental data exist on the coupling of the spin of the fundamental particles to gravitation. For this reason, in spite of the conceptual soundness of the teleparallel interpretation, we prefer to say that a definitive answer can only be achieved by further experiments.

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